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Virasoro algebra for particles with higher derivative interactions

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Abstract

In this Letter we show that the worldline reparametrization for particles with higher derivative interactions appears as a higher-dimensional symmetry, which is generated by a truncated Virasoro algebra. We also argue that for generic nonlocal particle theories the fields on the worldline may be promoted to those living on a two-dimensional worldsheet, and the reparametrization symmetry becomes locally the same as the conformal symmetry.

1. Introduction

Due to technical difficulties, physicists have been reluctant to consider higher derivative interactions. Yet they are unavoidable in many important physical problems. For a partial list, see [1]. In string theory, for example, we do not fully understand how to deal with higher derivative terms in the worldsheet action as they are nonrenormalizable, even though such background interactions should exist. String field theory is another outstanding example of nonlocal theory. For some recent interests in nonlocal theories in the context of string theory, see [2].

In fact, even for particles, higher derivative interactions are not well understood. In this Letter we study a very basic property of particle worldline theory—the reparametrization symmetry, for worldline Lagrangians with higher derivatives. We find that, remarkably, while the phase space of higher derivative theories are of higher dimensions, the reparametrization group acting on the phase space is also of higher dimension. Furthermore, for generic nonlocal theories, the reparametrization group is locally equivalent to the conformal group of two dimensions. It will be very interesting to see whether a new class of well-defined conformal field theories can originate from nonlocal worldline theories.

The plan of this Letter is as follows. We first review the worldline theory of a charged particle and consider its higher derivative generalization (Section 2). Then we show that both the phase space and the reparametrization group are of higher dimensions for higher derivative theories (Section 3). In the nonlocal limit, the reparametrization group becomes the conformal group (Section 4). Finally we comment on the connection to the string theory and the generalization of general covariance, as well as on the issue of stability (Section 5).

2. Worldline theory with higher derivatives

In this section, we begin by reviewing the particle worldline theory with only first time derivative.

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Then we consider the theory with second derivative, and show that there are more than one constraints corresponding to a larger symmetry. This motivates us to study the general case for the rest of this Letter.

2.1. First derivative

Let us start by reviewing the worldline theory of a charged particle moving in curved spacetime with metric $g_{\mu\nu}$

$$S_0 = \frac{1}{2}\int d\tau \left(e^{-1} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + A_\mu(x)\dot{x}^\mu - e\phi(x)\right), \quad (1)$$

where $e = e(\tau)$ is the tetrad of the worldline and ϕ is the potential term. For particles of mass m without external potential, $\phi = m^2$. The action is invariant under the reparametrization of the worldline parameter τ

$$\tau \to \tau' = \tau - \epsilon(\tau). \quad (2)$$

It induces a transformation of the worldline fields e and x

$$\delta e = \frac{d}{d\tau}(\epsilon e), \quad \text{and} \quad \delta x = \epsilon \dot{x}. \quad (3)$$

The symmetry of general coordinate transformation in the gravitational theory is reflected in the particle theory as the invariance of the action under a field redefinition $x^\mu \to x'^\mu(x)$ and a simultaneous transformation of the coefficients $g_{\mu\nu}$ and A_μ. The $U(1)$ symmetry of the gauge potential A_μ, on the other hand, appears as changing the Lagrangian by a total derivative.

The only generator of the reparametrization symmetry is the Hamiltonian H for this case. Variation of e in the action gives the constraint $H = 0$. In the usual covariant quantization scheme, we fix the gauge by setting

$$e = 1, \quad (4)$$

and impose the constraint $H = 0$ on the phase space of (x, P_x).

Another approach is not to fix the gauge, but to keep the full phase space of (e, x, P_x). The Poisson brackets on this phase space are defined by

$$(x, P_x) = 1, \qquad (e, x) = (e, P_x) = 0. \quad (5)$$

Again, a configuration is considered as a physical state only if it is annihilated by H. Although we seem to have one more variable e than the gauge-fixed description, it is not hard to check that this approach is equivalent to the former because the equation of motion for e implies that $\dot{e} = (e, H) = 0$. (If one wishes we can add the momentum P_e to the phase space and find one more constraint $P_e = \partial L/\partial \dot{e} = 0$ on the larger phase space of (e, P_e, x, P_x).)

For higher derivative theories, the gauge fixing (4) implies also that $\dot{e} = \ddot{e} = \cdots = 0$, and the only residual symmetry will be the translation of τ generated by H. Hence there will be only one constraint $H = 0$. The full reparametrization symmetry is not manifest in this approach.

Therefore, we will apply the latter approach (without gauge fixing) to higher derivative theories. We will find more than one constraints on the full phase space of $(e, x, P_x; P_e, \dot{x}, P_{\dot{x}}; \dot{e}, \ddot{x}, P_{\ddot{x}}; \ldots)$. Let us demonstrate this explicitly by the example below.

2.2. Second derivative

As the simplest example let us consider the following action with second time derivative

$$S = \int d\tau\, L = \int d\tau\, \frac{1}{2} e\left(D^2 x\right)^2, \quad (6)$$

where

$$D \equiv e^{-1}\frac{d}{d\tau} \quad (7)$$

is the covariant derivative of τ. The variation of S will be of the form

$$\delta S = \int d\tau \Big(\delta x\,(\mathrm{EOM})_x + \delta e\,(\mathrm{EOM})_e + \left(\delta x\, P_x + \delta\dot{x}\, P_{\dot{x}} + \delta e\, P_e\right)^{\cdot}\Big). \quad (8)$$

The terms $(\mathrm{EOM})_x$, $(\mathrm{EOM})_e$ are the equations of motion. The Hamiltonian is

$$H = \dot{e}P_e + \dot{x}P_x + \ddot{x}P_{\dot{x}} - L = e(Dx)P_x + \frac{1}{2}e^3 P_{\dot{x}}^2. \quad (9)$$

From the definition of P_e we find the constraint

$$\Phi \equiv P_e + (Dx)P_{\dot{x}} \simeq 0, \quad (10)$$

which induces a secondary constraint

$$(H, \Phi) = e^{-1}H \simeq 0. \quad (11)$$

If we impose the gauge fixing condition (4), then (4) and (10) become second class constraints while H is first class. The second class constraints define the Dirac bracket on the reduced phase space of $(x, \dot{x}, P_x, P_{\dot{x}})$, on which there is only one constraint (11).

In the other approach mentioned above, we do not fix the gauge and the full phase space is $(e, P_e, x, \dot{x}, P_x, P_{\dot{x}})$. We have the usual Poisson brackets defined on this phase space. Redefining the two constraints (10), (11) as

$$L_{-1} = -H, \qquad L_0 = -e\Phi, \tag{12}$$

we find the commutation relation

$$(L_0, L_{-1}) = L_{-1} \tag{13}$$

as suggested by our notation in the Virasoro algebra. From the commutation relations

$$(L_0, e) = e, \qquad (L_0, x) = 0, \qquad (L_0, \dot{x}) = \dot{x}, \tag{14}$$

$$(L_{-1}, e) = 0, \qquad (L_{-1}, x) = \dot{x},$$
$$(L_{-1}, \dot{x}) = \ddot{x} - e^{-1}\dot{e}\dot{x}, \tag{15}$$

and the equation of motion for e

$$\dot{e} = (e, H) = 0, \tag{16}$$

we see that L_{-1} and L_0 are the generators corresponding to the parameters ϵ and $\dot{\epsilon}$, respectively, for the reparametrization

$$\delta x = \epsilon\dot{x}, \qquad \delta\dot{x} = \epsilon\ddot{x} + \dot{\epsilon}\dot{x},$$
$$\delta e = \epsilon\dot{e} + \dot{\epsilon}e. \tag{17}$$

2.3. *Higher derivatives*

Let us now consider the worldline Lagrangian in its full generality. The most general particle Lagrangian $L(x, Dx, D^2x, \ldots; e)$ is an arbitrary function of x^μ and all of their covariant derivatives. We can expand L by the number of time derivatives as

$$\begin{aligned} L = e\Big[& A^{(0)}(x) + A^{(1)}_\mu(x)Dx^\mu \\ & + \left(A^{(01)}_\mu(x)D^2x^\mu + \frac{1}{2}A^{(20)}_{\mu\nu}(x)Dx^\mu Dx^\nu\right) \\ & + \big(A^{(001)}_\mu D^3x^\mu + A^{(110)}_{\mu\nu}D^2x^\mu Dx^\nu \\ & + A^{(300)}_{\mu\nu\lambda}Dx^\mu Dx^\nu Dx^\lambda\big) + \cdots\Big]. \end{aligned} \tag{18}$$

For a given order n of D and a set of integers $P(n) = \{P_k \geqslant 0; k = 1, \ldots, n\}$ satisfying $\sum_{k=1}^n kP_k = n$, there is a spacetime field $A^{(P_1,\ldots,P_n)}(x)$ coupled to the particle by the interaction

$$\begin{aligned} L_{P(n)} &\equiv eA^{P(n)} \\ &= e\left(A^{P(n)}_{\mu_1\ldots\mu_m}\prod_{i=1}^{P_1}Dx^{\mu_i}\prod_{j=1}^{P_2}D^2x^{\mu_{P_1+j}} \right.\\ &\qquad \left.\times\prod_{k=1}^{P_3}D^3x^{\mu_{P_1+P_2+k}}\cdots\right) \end{aligned} \tag{19}$$

invariant under (3), where $m = \sum_{k=1}^n P_k$.

The variation of S gives the definition of conjugate momenta for $e^{(n)}$ and $x^{(n)}$. We define ordinary Poisson brackets on the full phase space of $(\{e^{(n)}, P_{e^{(n)}}, x^{(n)}, P_{x^{(n)}}\})$, and there will be primary constraints for each $P_{e^{(n)}}$, which induce the secondary constraints.

Our example in Section 2.2 suggests that these constraints correspond to the reparametrization symmetry. The variation of the action due to reparametrization is of the form

$$\begin{aligned} \delta S &= \int_{t_0}^{t_1} d\tau\Big[\sum_n\big(\delta e^{(n)}P_{e^{(n)}} + \delta x^{(n)}P_{x^{(n)}}\big)^{\cdot} \\ &\qquad + \delta x(\mathrm{EOM})_x + \delta e(\mathrm{EOM})_e\Big] \\ &= \Big[\sum_n\epsilon^{(n)}(t)Q_n\Big]_{t_0}^{t_1}, \end{aligned} \tag{20}$$

where we assume that the equation of motion is satisfied. This has to be zero for any $\epsilon(t)$, and so the generators of reparametrization Q_n have to vanish. Therefore, the definitions of $P_{e^{(n)}}$ together with the equations of motion have to guarantee that $Q_n = 0$. Conversely, (at least some of) the primary and secondary constraints can be understood as the generators of reparametrization.

3. Reparametrization symmetry

We start with Lagrangians (18) involving only finite order time derivatives. Assume that the Lagrangian (18) is a function of $\{x^{(n)};\ n = 0, 1, \ldots, N\}$, where we used the notation $f^{(n)} \equiv (d/d\tau)^n f(\tau)$, and that it is nondegenerate. To apply canonical quantization, we can introduce new variables $x_1, \ldots, x_{N-1}$ and add

$$L' = \sum_{i=1}^{N-1} \lambda_i (x_i - \dot{x}_{i-1}) \tag{21}$$

to the Lagrangian, where λ_i's are the Lagrange multipliers. This trick allows us to replace $x^{(n)}$ by x_n for $n = 1, 2, \ldots, N-1$ in the Lagrangian, which then becomes a function of $x_0, x_1, \ldots, x_{N-1}$ and $\dot{x}_{N-1}$, with only first time derivatives. Dirac's constrained quantization can be applied straightforwardly.

The equations of motion of the x's are differential equations of order $2N$, which require $2N$ initial data to determine a solution. One can think of the phase space as the space of initial conditions given at $\tau = 0$. Hence the phase space for each x is $2N$-dimensional.

The fact that we have a larger phase space for higher derivative theories has significant implications. Consider the worldline reparametrization symmetry (3). Taking its kth derivative and evaluating it at $\tau = 0$, we get

$$\delta x^{\mu(k)}(0) = \sum_{m=0}^{k} C^k_m \epsilon^{(m)}(0) x^{\mu(k-m+1)}(0),$$
$$\text{where} \quad C^n_m = \frac{n!}{m!(n-m)!}. \tag{22}$$

Although these expressions follow directly from (3), they are independent transformations on the phase space for $k = 0, 1, 2, \ldots, N-1$, since x, $\dot{x}$, etc., are independent variables. Eq. (22) defines N independent transformations with parameters $\epsilon(0), \dot{\epsilon}(0), \ldots, \epsilon^{(N-1)}(0)$.

Note that for (22), as well as any other expression for x in this section, there is an analogous one for e. But we will omit them here for brevity because they can be easily obtained in a similar fashion.

Naively, even for $k \geqslant N$, Eq. (22) still looks like a transformation on the phase space. But it is inconsistent to treat them as independent transformations because they do not preserve the Poisson structure. In other words, it is impossible to find operators to generate these transformations.

The above can be derived more rigorously as follows. We add (21) to the Lagrangian and replace $x^{(n)}$'s by x_n's. The Lagrangian now has only first derivatives of x. In order for L' to be invariant under the reparametrization, we need

$$\delta\lambda_k = \frac{d}{d\tau}\left[\sum_{m=0}^{N-k-1} C^{k+m}_{m+1} \epsilon^{(m)} \lambda_{k+m}\right],$$
$$\delta x_k = \sum_{m=0}^{k} C^k_m \epsilon^{(m)} \dot{x}_{k-m}. \tag{23}$$

The 2nd transformation law in (23) agrees with (22) and can be conveniently summarized as

$$\delta x(t+\sigma) = \epsilon(t+\sigma)\dot{x}(t+\sigma), \tag{24}$$

where we only need the first N terms in the Taylor expansion

$$x(\tau+\sigma) = \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} x_n(\tau),$$
$$\epsilon(\tau+\sigma) = \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \epsilon^{(n)}(\tau). \tag{25}$$

Note that λ_n is identified with p_{n-1}, the conjugate momentum of $x^{(n-1)}$, in the constrained quantization. So (23) also tells us how the momenta transforms.

The N symmetry generators corresponding to $\epsilon^{(k)}$ are

$$H_k \equiv \sum_{m=k}^{N-1} C^m_k x^{\mu(m-k+1)} p_{\mu(m)}$$
$$\text{for } k = 0, \ldots, N-1. \tag{26}$$

We see that the reparametrization symmetry is N-dimensional for a particle theory with Nth time derivatives.

A redefinition of the H_k's (26) to $L_k \equiv i(k+1)! \times H_{k+1}$ ($k = -1, 0, 1, \ldots, N-2$) leads to

$$[L_m, L_n] = \begin{cases} (m-n)L_{m+n} \\ \quad \text{for } m+n \leqslant N-2, \\ 0 \quad \text{for } m+n > N-2, \end{cases} \tag{27}$$

for $m, n = -1, 0, 1, \ldots, N-2$. Note that this is a simple truncation of the classical Virasoro algebra in the following sense. We have a set of consistent commutation relations (27) which can be fully identified with a subset of those in the Virasoro algebra by assigning $L_n = 0$ for $n > N-2$ on the right-hand side. This is one of the main results of this Letter.

Roughly speaking, in the $N \to \infty$ limit, we expect to obtain only "half" of the Virasoro algebra. In order to have the full algebra, one has to consider nonlocal theories which are not defined as the large N limit of finite derivative theories.

4. Conformal symmetry for nonlocal particle

In this section we consider generic nonlocal particle theories. As various kinds of nonlocal theories may differ significantly in nature, in order for a generic discussion without specifying details of the theory, this section will be more heuristic and less rigorous than before. We do not expect everything in this section to hold for *all* nonlocal theories, but we believe that our results will apply to a wide class of examples. In fact, it will be very interesting to have *any* nonlocal particle theory for which the reparametrization symmetry coincides with the conformal symmetry.

To deal with Lagrangians with infinite derivatives, an interesting proposal [3–5] is to introduce an auxiliary coordinate σ as follows. For $x(\tau)$ we introduce $X(\tau,\sigma)$ and impose the constraint

$$\dot{X}(\tau,\sigma) = X'(\tau,\sigma), \tag{28}$$

where $X' \equiv \frac{d}{d\sigma}X$. This guarantees that X can be identified with the original variable as $X(\tau,\sigma) = x(\tau+\sigma)$. Since the generic action (18) also involves higher derivatives of the tetrad $e(\tau)$, it should also be promoted to $E(\tau,\sigma)$, satisfying a constraint like (28). The covariant derivative D (7) will be replaced by $D_\sigma = E^{-1}(\tau,\sigma)\partial_\sigma$ when acting on X. By replacing all D by D_σ and $x(\tau)$ by $X(\tau,\sigma)$ in L_0, we obtain a new Lagrangian L_1 which has the same equation of motion if the constraint (28) is satisfied.

The constraint (28) can be imposed by adding

$$L' = \int d\sigma\,\lambda(\sigma)\big(X'(\sigma) - \dot{X}(\sigma)\big) \tag{29}$$

to L_1. This can also be obtained as an $N \to \infty$ limit of (21). The new Lagrangian $L \equiv L_1 + L'$ does not have any higher time derivatives, but only higher derivatives of the auxiliary coordinate σ.

The worldline reparametrization (2) of the original Lagrangian L_0 induces the reparametrization of σ

$$\delta\sigma = -\epsilon(\tau+\sigma), \tag{30}$$

leaving τ invariant. The derivative D_σ and the scalar field X should be invariant under (30), that is, $f(\tau,\sigma) = (f+\delta f)(\tau,\sigma+\delta\sigma)$ for $f = D_\sigma$ and X. It follows that the transformation of $E(\tau,\sigma)$ and $X(\tau,\sigma)$ are given by

$$\delta E(\tau,\sigma) = \partial_\sigma\big(\epsilon(\tau+\sigma)E(\tau,\sigma)\big), \tag{31}$$

$$\delta X(\tau,\sigma) = \epsilon(\tau+\sigma)\partial_\sigma X(\tau,\sigma). \tag{32}$$

To have L_1 invariant under (30), we also need the measure of integration $E(\tau,\sigma)\,d\tau \wedge d\sigma$ to be invariant, which can be easily verified.

For L' to be invariant, we need

$$\delta\lambda(\tau,\sigma) = \partial_\sigma\big(\epsilon(\tau+\sigma)\lambda(\tau,\sigma)\big). \tag{33}$$

In addition, we need to impose certain boundary conditions at the boundary values of σ (denoted σ_0, σ_1) such that

$$\lambda(\tau,\sigma)(X' - \dot{X})\big|_{\sigma_0}^{\sigma_1} = 0. \tag{34}$$

The boundary condition can be chosen to be

$$\lambda(\tau,\sigma) = 0, \quad \text{for } \sigma = \sigma_0, \sigma_1,\ \forall\tau. \tag{35}$$

This is also an appropriate condition to guarantee that the equation of motion for the new Lagrangian L is equivalent to the original equation of motion. By varying X and λ in the new Lagrangian L, we get (28) and

$$\dot{\lambda} - \lambda' = (\text{EOM}), \tag{36}$$

where (EOM) is the expression for the equation of motion for the original Lagrangian L_0, but with x replaced by X, etc. Using (28) we can replace $X(\tau,\sigma)$ by $x(\tau+\sigma)$, but (36) does not guarantee that (EOM) $= 0$. Instead, because (EOM) is a function of $(\tau+\sigma)$ only, (36) implies that

$$\lambda(\tau,\sigma) = (\tau-\sigma)(\text{EOM}) + f(\tau+\sigma) \tag{37}$$

for an arbitrary function f. Applying the boundary condition (35) to it, we find two identities valid for all τ. They imply $\lambda = 0$ and (EOM) $= 0$ as we hoped.

As the reparametrization symmetry (30) is labelled by a one-variable function ϵ at $\tau = 0$, it is locally the same as the conformal symmetry in two dimensions.

So, far we have restricted ourselves to the classical theory. In principle, the quantum anomaly (the central charge of the Virasoro algebra) can appear in the large N limit, but we need further details of the theory in order to treat it with some rigor.

5. Remarks

5.1. Connection to string theory

Naively, higher derivative particle Lagrangians can be related to the boundary string field theory [6] as the open string worldsheet Lagrangian in the zero metric limit. If this interpretation is acceptable, the boundary string field theory can be interpreted as the theory over "the space of all particle theories" instead of "the space of all open string boundary theories". One can also view strings as a convenient technique to summarize the infinite degrees of freedom of particles with nonlocal interactions. However, it is hard to make this story rigorous due to the technical problem that higher derivative terms in the string worldsheet action are nonrenormalizable.

Another possible connection with string theory is to generalize the Seiberg–Witten (SW) limit [7] for open strings ending on D-branes in a constant B field background. Since the open string vertex operator for the $U(1)$ field interaction involves only the first time derivative, only the zero mode of the string survives in the SW limit. If other open string vertex operators with higher derivatives are considered, more degrees of freedom of the string will survive the analogous SW limit, if it exists.

5.2. Symmetry of spacetime fields

In the infinite derivative limit, we can generalize the story of general covariance and $U(1)$ symmetry mentioned in the paragraph below Eq. (3) in Section 2. The particle Lagrangian (18) is invariant under a simultaneous worldline field redefinition

$$x \to x' = x'(x, \dot{x}, \ddot{x}, \ldots) \tag{38}$$

and a complicated, mixed transformation of the fields $A^{(P)}$. This defines a generalized notion of general covariance. Similar transformations for the string coordinates has been used to construct string field theory [8], and it would be interesting to see if they have any connection with the higher spin gauge theories [9].

Generalization of the $U(1)$ symmetry is straightforward. The Lagrangian is always defined only up to a total derivative $\frac{d}{d\tau}\Lambda(x, \dot{x}, \ddot{x}, \ldots)$, which induces a gauge transformation of the $A^{P(n)}$'s.

5.3. Stability problem

Higher-derivative theories are known to have various problems such as stability and causality. Since the notion of causality may be changed at the Planck scale where spacetime is fuzzy, we will only comment on the problem of stability. Theories with higher derivatives of finite order suffer the Ostrogradskian instability because the canonical Hamiltonian is always unbounded from below [10]. For low energy effective theories, the perturbative formulation [1] might be a convenient way to avoid these problems. On the other hand, stable nonlocal theories are known to exist. At least, those theories obtained from integrating out certain physical fields of a stable field theory should still be stable.

In this Letter we studied worldline theories with higher derivatives. Although the Hamiltonian is constrained to vanish, and thus is not unbounded from below, an attempt to couple it to other physical systems may still lead to instability. Ideally, this work should have been focused on nonlocal theories without instability. We hope this Letter may serve as part of the motivation for the task of understanding how to construct sensible nonlocal theories.

Note added in revision

After the first version of this Letter appeared, two earlier works [11,12] were brought to my attention. These works considered nonlocal particle Lagrangians closely related to the explicit example of this Letter. The work in Ref. [11] did not mention reparametrization symmetry, and their model is not exactly the same as the string. In the other work [12], a quantization different from the canonical quantization of the original variables was chosen such that it is equivalent to the open string. It was also argued there that the reparametrization symmetry is equivalent to the conformal symmetry for quadratic Lagrangians. In this Letter, we have provided a comparatively more general and more explicit discussion on the reparametrization symmetry.

I was also informed that, in a series of papers [13], generic particle Lagrangians with only first derivatives are considered as the starting point of formulating the “generalized equivalence principle”. Our proposal for “generalized general covariance” described above is simply a generalization of that to include higher derivatives. This is a crucial difference when comparing the corresponding higher spin gauge theory with string theory. Only totally symmetrized tensor fields appear in the first derivative theory of [13], but we know that in string theory there are many more tensor fields. Our model contains all the tensor field degrees of freedom in open string theory.

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